available decorative lamp with fine plastic fibres with their free ends forming a fountain like structure. The other end of the fibres is fixed over an electric lamp. When the lamp is switched on, the light travels from the bottom of each fibre and appears at the tip of its free end as a dot of light. The fibres in such decorative lamps are optical fibres.

The main requirement in fabricating optical fibres is that there should be very little absorption of light as it travels for long distances inside them. This has been achieved by purification and special preparation of materials such as quartz. In silica glass fibres, it is possible to transmit more than 95% of the light over a fibre length of 1 km. (Compare with what you expect for a block of ordinary window glass 1 km thick.)

9.5 Refraction at Spherical Surfaces and by Lenses

We have so far considered refraction at a plane interface. We shall now consider refraction at a spherical interface between two transparent media. An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface. Just as for reflection by a spherical mirror, the normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point and, therefore, passes through its centre of curvature. We first consider refraction by a single spherical surface and follow it by thin lenses. A thin lens is a transparent optical medium bounded by two surfaces; at least one of which should be spherical. Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, we shall obtain the lens maker's formula and then the lens formula.

9.5.1 Refraction at a spherical surface

Figure 9.17 shows the geometry of formation of image *I* of an object *O* on the principal axis of a spherical surface with centre of curvature C, and radius of curvature *R*. The rays are incident from a medium of refractive index n_1, to another of refractive index n_2. As before, we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. In particular, NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis. We have, for small angles,

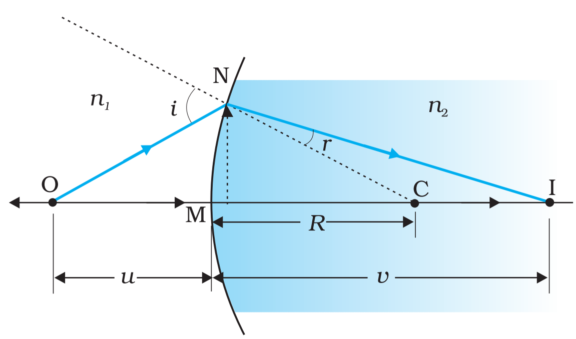

FIGURE 9.17 Refraction at a spherical surface separating two media.

$$\tan \angle NOM = \frac{MN}{OM}$$

$$\tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

LIGHT SOURCES AND PHOTOMETRY

It is known that a body above absolute zero temperature emits electromagnetic radiation. The wavelength region in which the body emits the radiation depends on its absolute temperature. Radiation emitted by a hot body, for example, a tungsten filament lamp having temperature 2850 K are partly invisible and mostly in infrared (or heat) region. As the temperature of the body increases radiation emitted by it is in visible region. The sun with temperature of about 5500 K emits radiation whose energy versus wavelength graph peaks approximately at 550 nm corresponding to green light and is almost in the middle of the visible region. The energy versus wavelength distribution graph for a given body peaks at some wavelength, which is inversely proportional to the absolute temperature of that body.

The measurement of light as perceived by human eye is called *photometry*. Photometry is measurement of a physiological phenomenon, being the stimulus of light as received by the human eye, transmitted by the optic nerves and analysed by the brain. The main physical quantities in photometry are (i) the *luminous intensity* of the source, (ii) the *luminous flux* or flow of light from the source, and (iii) *illuminance* of the surface. The SI unit of *luminous intensity* (I) is candela (cd). The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian. If a light source emits one candela of luminous intensity into a solid angle of one steradian, the total luminous flux emitted into that solid angle is one *lumen* (lm). A standard 100 watt incadescent light bulb emits approximately 1700 lumens.

In photometry, the only parameter, which can be measured directly is *illuminance*. It is defined as luminous flux incident per unit area on a surface (lm/m^2 or *lux*). Most light meters measure this quantity. The illuminance E, produced by a source of luminous intensity I, is given by $E = I/r^2$, where r is the normal distance of the surface from the source. A quantity named *luminance* (L), is used to characterise the brightness of emitting or reflecting flat surfaces. Its unit is cd/m^2 (sometimes called 'nit' in industry). A good LCD computer monitor has a brightness of about 250 nits.

Now, for ΔNOC, i is the exterior angle. Therefore, $i = \angle NOM + \angle NCM$

$$i = \frac{MN}{OM} + \frac{MN}{MC} \tag{9.13}$$

Similarly,

$$r = \angle NCM - \angle NIM$$

$$\text{i.e., } r = \frac{MN}{MC} - \frac{MN}{MI} \tag{9.14}$$

Now, by Snell's law

$$n_1 \sin i = n_2 \sin r$$

or for small angles

$$n_1 i = n_2 r$$

Substituting *i* and *r* from Eqs. (9.13) and (9.14), we get

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \tag{9.15}$$

Here, OM, MI and MC represent magnitudes of distances. Applying the Cartesian sign convention,

OM = $-u$, MI = $+v$, MC = $+R$

Substituting these in Eq. (9.15), we get

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \tag{9.16}$$

Equation (9.16) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface.

Example 9.6 Light from a point source in air falls on a spherical glass surface (n = 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

Solution

We use the relation given by Eq. (9.16). Here
u = – 100 cm, v = ?, R = + 20 cm, n_1 = 1, and n_2 = 1.5.
We then have

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20}$$

or v = +100 cm

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

EXAMPLE 9.6

9.5.2 Refraction by a lens

Figure 9.18(a) shows the geometry of image formation by a double convex lens. The image formation can be seen in terms of two steps: (i) The first refracting surface forms the image I_1 of the object O [Fig. 9.18(b)]. The image I_1 acts as a virtual object for the second surface that forms the image at I [Fig. 9.18(c)]. Applying Eq. (9.15) to the first interface ABC, we get

$$\frac{n_1}{OB} + \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} \tag{9.17}$$

A similar procedure applied to the second interface* ADC gives,

$$-\frac{n_2}{DI_1} + \frac{n_1}{DI} = \frac{n_2 - n_1}{DC_2} \tag{9.18}$$

* Note that now the refractive index of the medium on the right side of ADC is n_1 while on its left it is n_2. Further DI_1 is negative as the distance is measured against the direction of incident light.

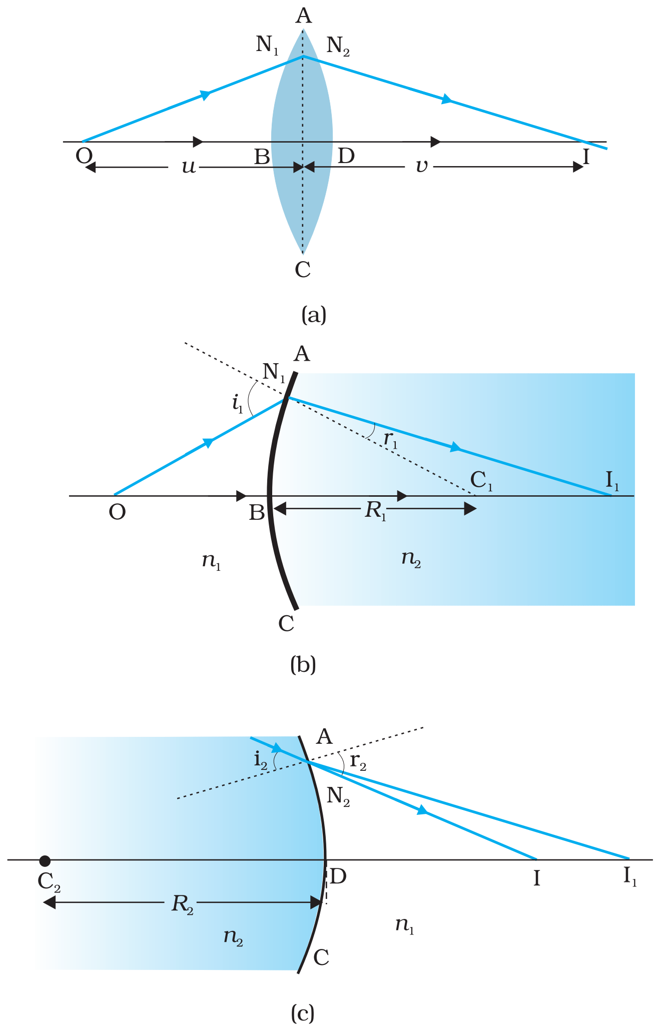

FIGURE 9.18 (a) The position of object, and the image formed by a double convex lens, (b) Refraction at the first spherical surface and (c) Refraction at the second spherical surface.

For a thin lens, $BI_1 = DI_1$. Adding Eqs. (9.17) and (9.18), we get

$$\frac{n_1}{OB}+\frac{n_1}{DI}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \tag{9.19}$$

Suppose the object is at infinity, i.e., $OB \to \infty$ and $DI = f$, Eq. (9.19) gives

$$\frac{n_1}{f}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \tag{9.20}$$

The point where image of an object placed at infinity is formed is called the *focus* F, of the lens and the distance *f* gives its *focal length*. A lens has two foci, F and F′, on either side of it (Fig. 9.19). By the sign convention,

$BC_1 = + R_1$,

$DC_2 = -R_2$

So Eq. (9.20) can be written as

$$\frac{1}{f}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \left(\because n_{21}=\frac{n_2}{n_1}\right) \tag{9.21}$$

Equation (9.21) is known as the *lens maker's formula*. It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature. Note that the formula is true for a concave lens also. In that case R_1 is negative, R_2 positive and therefore, *f* is negative.

From Eqs. (9.19) and (9.20), we get

$$\frac{n_1}{OB}+\frac{n_1}{DI}=\frac{n_1}{f} \tag{9.22}$$

Again, in the thin lens approximation, B and D are both close to the optical centre of the lens. Applying the sign convention,

$BO = -u$, $DI = +v$, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \tag{9.23}$$

Equation (9.23) is the familiar *thin lens formula*. Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lenses and for both real and virtual images.

It is worth mentioning that the two foci, F and F′, of a double convex or concave lens are equidistant from the optical centre. The focus on the side of the (original) source of light is called the *first focal point*, whereas the other is called the *second focal point*.

To find the image of an object by a lens, we can, in principle, take any two rays emanating from a point on an object; trace their paths using the laws of refraction and find the point where the refracted rays meet (or appear to meet). In practice, however, it is convenient to choose any two of the following rays:

(i) A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F′ (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F.

(ii) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.

(iii) (a) A ray of light passing through the first principal focus of a convex lens [Fig. 9.19(a)] emerges parallel to the principal axis after refraction.

(b) A ray of light incident on a concave lens appearing to meet the principal axis at second focus point emerges parallel to the principal axis after refraction [Fig. 9.19(b)].

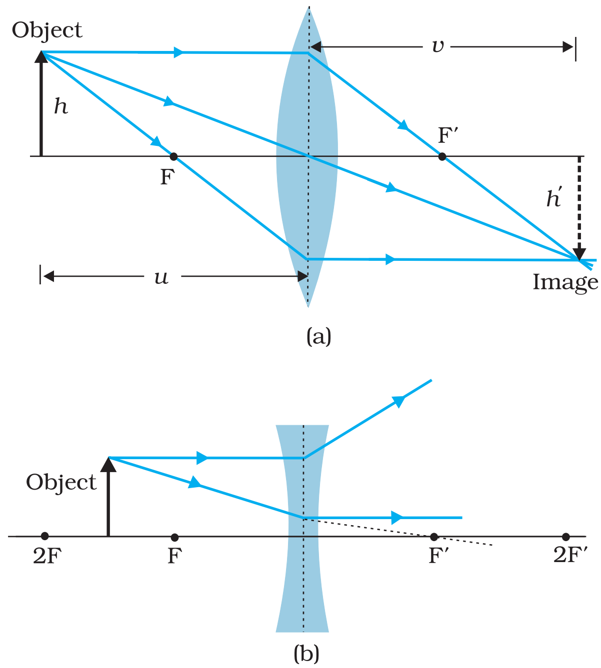

FIGURE 9.19 Tracing rays through (a) convex lens (b) concave lens.

Figures 9.19(a) and (b) illustrate these rules for a convex and a concave lens, respectively. You should practice drawing similar ray diagrams for different positions of the object with respect to the lens and also verify that the lens formula, Eq. (9.23), holds good for all cases.

Here again it must be remembered that each point on an object gives out infinite number of rays. All these rays will pass through the same image point after refraction at the lens.

Magnification (m) produced by a lens is defined, like that for a mirror, as the ratio of the size of the image to that of the object. Proceeding in the same way as for spherical mirrors, it is easily seen that for a lens

$$m = \frac{h'}{h} = \frac{v}{u} \qquad (9.24)$$

When we apply the sign convention, we see that, for erect (and virtual) image formed by a convex or concave lens, m is positive, while for an inverted (and real) image, m is negative.

Example 9.7 A magician during a show makes a glass lens with n = 1.47 disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

Solution

The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives $1/f = 0$ or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

EXAMPLE 9.7

9.5.3 Power of a lens

Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it. Clearly, a lens of shorter focal

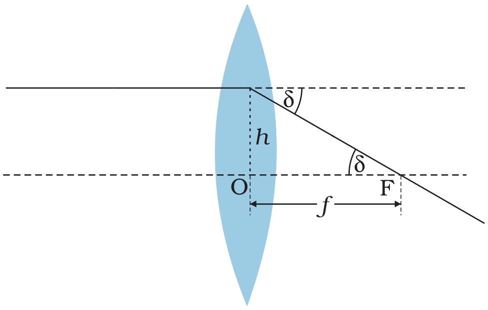

FIGURE 9.20 Power of a lens.

length bends the incident light more, while converging it in case of a convex lens and diverging it in case of a concave lens. The *power P* of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light parallel to the principal axis falling at unit distance from the optical centre (Fig. 9.20).

$$\tan\delta = \frac{h}{f};\ \text{if } h = 1,\quad \tan\delta = \frac{1}{f}\quad \text{or}\quad \delta = \frac{1}{f}\ \text{for small}$$

value of δ. Thus,

$$P = \frac{1}{f} \tag{9.25}$$

The SI unit for power of a lens is dioptre (D): 1D = 1m^{-1}. The power of a lens of focal length of 1 metre is one dioptre. Power of a lens is positive for a converging lens and negative for a diverging lens. Thus, when an optician prescribes a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length + 40 cm. A lens of power of – 4.0 D means a concave lens of focal length – 25 cm.

Example 9.8 (i) If f = 0.5 m for a glass lens, what is the power of the lens? (ii) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass? (iii) A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)

Solution

(i) Power = +2 dioptre.

(ii) Here, we have f = +12 cm, R_1 = +10 cm, R_2 = –15 cm.
Refractive index of air is taken as unity.
We use the lens formula of Eq. (9.22). The sign convention has to be applied for f, R_1 and R_2.
Substituting the values, we have

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$

This gives n = 1.5.

(iii) For a glass lens in air, n_2 = 1.5, n_1 = 1, f = +20 cm. Hence, the lens formula gives

$$\frac{1}{20} = 0.5\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For the same glass lens in water, n_2 = 1.5, n_1 = 1.33. Therefore,

$$\frac{1.33}{f} = (1.5 - 1.33)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{9.26}$$

Combining these two equations, we find f = + 78.2 cm.

EXAMPLE 9.8

9.5.4 Combination of thin lenses in contact

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. Let the object be placed at a point O beyond the focus of the first lens A (Fig. 9.21). The first lens produces an image at I_1. Since image I_1 is real, it serves as a virtual object for the second lens B, producing the final image at I. It must, however, be borne in mind that formation of image by the first lens is presumed only to facilitate determination of the position of the final image. In fact, the direction of rays emerging from the first lens gets modified in accordance with the angle at which they strike the second lens. Since the lenses are thin, we assume the optical centres of the lenses to be coincident. Let this central point be denoted by P.

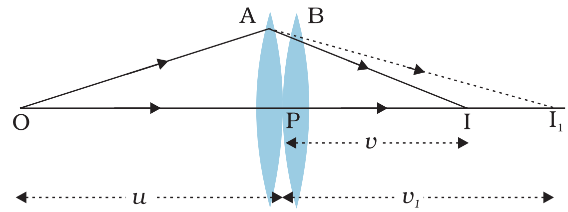

FIGURE 9.21 Image formation by a combination of two thin lenses in contact.

For the image formed by the first lens A, we get

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \qquad (9.27)$$

For the image formed by the second lens B, we get

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \qquad (9.28)$$

Adding Eqs. (9.27) and (9.28), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (9.29)$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

so that we get

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (9.30)$$

The derivation is valid for any number of thin lenses in contact. If several thin lenses of focal length $f_1, f_2, f_3,...$ are in contact, the effective focal length of their combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots \qquad (9.31)$$

In terms of power, Eq. (9.31) can be written as

$$P = P_1 + P_2 + P_3 + \ldots \qquad (9.32)$$

where P is the net power of the lens combination. Note that the sum in Eq. (9.32) is an algebraic sum of individual powers, so some of the terms on the right side may be positive (for convex lenses) and some negative (for concave lenses). Combination of lenses helps to obtain diverging or converging lenses of desired magnification. It also enhances sharpness of the image. Since the image formed by the first lens becomes the object for the second, Eq. (9.25) implies that the total magnification m of the combination is a product of magnification (m_1, m_2, m_3,...) of individual lenses

$$m = m_1 m_2 m_3 \ \ldots \qquad (9.33)$$