CH. Differential equations (D.E)

D.C coefficients (D.C)

$\frac{dy}{dx} \longrightarrow$ Ist order D.C

$\frac{d^2y}{dx^2} \longrightarrow 2^{nd}$ order D.C

$\vdots$

$\frac{d^ny}{dx^n} \longrightarrow n^{th}$ order D.C

DE:- Equations involving atleast one D.C.

Order of D.E :- (Always defined) :- Highest order of the D.C^s involved in D.E.

Degree of D.E :- If D.E can be written ( ) polynomial eq^n of all involved D.C^s then the power (degree) of the highest order D.C^s is called degree of D.E. If all involved D.C^s can NOT be written in poly. form then degree of D.E is NOT defined.

Q.(1)	D.E	Order	Degree
(ii)	$\sqrt{\frac{dy}{dx}} = \sin x$ इसको हम polynomial form में लिख सकते हैं। Like $\frac{dy}{dx} = \sin^2 x$ ⇒	1	1.
(iv) **	$\frac{d^2y}{dx^2} = \left(y + \left(\frac{dy}{dx}\right)^6\right)^{1/4}$	2	4
(v)	$\frac{dy}{dx} + y = \frac{1}{\left(\frac{dy}{dx}\right)}$ (Cross multiply)	1	2
(vi)	$\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$ tak sin both the sides $\frac{dy}{dx} = \sin(x+y)$	1	1
Q.2 (i)	$e^{\frac{d^3y}{dx^3}} - x\frac{d^2y}{dx^2} + y = 0$	3	N.D Eq^n cannot be expressed as in form of polynomial
(ii)	$\ln\left(\frac{dy}{dx}\right) = ax + by$	1	1

Note :- If D.E contains radicals or fractional power on D.C^s then in order to define degree of D.E, minimum power must be raised to convert D.E in polynomial eq^n of D.C^s.

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Q. $\frac{d^2y}{dx^2} = \sin\left(\frac{dy}{dx}\right) \Rightarrow$ order $= 2$ NOT. Defined.

⇒ Independent Arbitrary constants (I.A.C).

Minimum no. of arbitrary constants required to represent any equation are called I.A.C.

Q.5) i) Find I.A.C for $y = \underbrace{(c_1+c_2)}_{A} \cos(x+c_3) - c_4 e^{x+c_5}$

Ans.

$c_4 e^{x+c_5} \rightarrow B \leftarrow c_4 e^{c_5} e^x$

$y = A(\cos(x+c_3)) - Be^x$

$\Rightarrow$ I.A.C $= 3$

$\boxed{\text{Note}}$:- No. of IAC of any curve will be the order of D.E of that curve.

⇒ **Formation of D.E of curve from given eq^n of curve:-**

step (1) :- Find I.A.C i.e order of D.E ; let IAC is 'n' ; $n \in N$

step (2) :- Differentiate given eq^n of curve 'n' times.

step (3) :- Eleminate all I.A.C to get the final D.E.

$\boxed{\text{Note}}$:- A D.E represents family of curves all satisfying some common properties. This can be considered as geometrical interpretation of the differential equations.

Q.7) Find D.E of family of lines concurrent at origin?

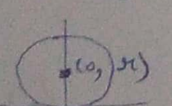
Ans

$y = mx + \cancel{c}_0 \Rightarrow \frac{y}{x} = m$

$\Rightarrow$ I.A.C $= 1$ (one) $\Rightarrow y' = m \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

Q.9) Find D.E of all circles touching x-axis at origin & centre on y-axis.

Ans.



$(x-h)^2 + (y-g)^2 = r^2$

$\rightarrow (x)^2 + (y-r)^2 = r^2$

$\Rightarrow$ I.A.C $= 1$ (one)

$x^2 + y^2 + \cancel{r^2} - 2yr = \cancel{r^2}$

$x^2 + y^2 = 2yr \rightarrow \frac{x^2+y^2}{2y} = r$

$2x + \cancel{2}yy' = \cancel{2}ry'$

$y' = -\frac{x}{(y-r)}$

$y' = \frac{x}{y - \left(\frac{x^2+y^2}{2y}\right)} = \frac{2xy}{-x^2+y^2}$

* (Note that यहाँ पे r और r^2 दो independent arbitrary constants नहि ⇒ I.A.C ≠ 2 यहाँ पे r^2, r कि value पे depend कर रहा है।)

Q.11) The D.E of the curve $y^2 = 2c(x+\sqrt{c})$ will have :-

(A) Order = 2, Degree = 3   (B) order = 1 ; degree = 3
(C) order = 1, Degree Not defined   (D) None.

Ans.

$$y^2 = 2c\,(x+\sqrt{c}) \quad \text{--- (1)}$$

I.A.C → (1 one) ★

$$2yy' = 2c\,(1)$$

$$y' = \frac{c}{y} \quad \text{--- (2)}$$

(यहाँ से c कस रहा है हम यहाँ से c की value नहि निकाल पा रहे हैं तो क्या हुआ। हम eqⁿ 2 से c की value substitute कर देंगे eqⁿ - (1) में)

$$\Rightarrow y^2 = \cancel{2}yy'\,(x + \sqrt{yy'})$$

$$y^2 = yy' + (yy')^{3/2} \quad \text{(square on both sides)}$$

$$\Rightarrow (y^2 - yy')^2 = (yy')^3$$

correct option → (B) Ans.

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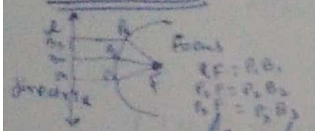
extra point

Parabola :-



A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

The fixed line is called the directrix of the parabola and the fixed point F is called the focus. ('Para' means 'for' & bola means 'throwing', i.e, the shape described when you throw a ball in the air).

★ → Parabola is symmetric w.r.to axis of the parabola. If the eqⁿ has a y^2 term, then the axis of symmetry is along x-axis & if the eqⁿ has an x^2 term, then the axis of symmetry is along y-axis.

Q.13] Find D.E of curve $y = c_1 e^{3x} + c_2 e^{2x} + c_3 e^{x}$; c_1, c_2, c_3 are A.C's.

Ans. I.A.C ⟹ 3

$$y = c_1 e^{3x} + c_2 e^{2x} + c_3 e^{x}$$

★ $$\frac{y}{e^x} = c_1 e^{2x} + c_2 e^{x} + c_3 \xrightarrow{\text{diff}} -ye^{-x} + e^{-x}y' = 2c_1 e^{2x} + c_2 e^{x} + 0$$

diff then again multiply by e^{-x} then diff to get final D.E.

➡ Solution of D.E's

① general Solⁿ

$$y = f(x) + c$$

② Particular solⁿ.
For a given condⁿ find value of 'c' finally $y = f(x)$.

→ Solving Different types of D.E's :-

Type ① :- (A) Direct variable Separable :-
If D.E can be written as $f(y)\,dy = g(x)\,dx$ then $\int f(y)\,dy = \int g(x)\,dx$ to solve D.E (i.e separate y's & x's)

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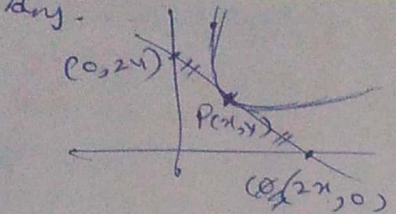
Q. ? for $x \cdot y = k$; segment of every tangent contained b/w ~~at~~ axes is bisected at point of tangency and area of the Δ formed by tangent and axes is $2k$ where $k > 0$?

Ans.



$$\frac{dy}{dx} = \cancel{xy' + y} = \frac{2y - 0}{0 - 2x} \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \ln y = -\ln x + c$$

$$\boxed{xy = k}$$

$$\Delta = \frac{1}{2}(2x)(2y) = 2xy \Rightarrow \Delta = 2k \text{ Ans}$$

Q. 20] The sol^n of equation $\int_0^1 y(xt)\,dt = n\,y(x)$ is ?

Ans.

$$\int_0^1 y(xt)\,dt = n\,y(x)$$

$\longrightarrow$ $\int_0^1 f(xt)\,dt = n f(x)$ → here t is variable.

Put $\boxed{xt = z} \Rightarrow x\,dt = dz$ $\qquad t \to 0 \Rightarrow z \to 0$, $\; t \to 1 \Rightarrow z \to x$

$$\frac{1}{x}\int_0^x f(z)\,dz = n f(x)$$

$$\int_0^x f(z)\,dz = n x f(x) \xrightarrow{\text{diff}} f(x) = n\left(f(x) + x f'(x)\right)$$

$$(1-n) f(x) = x n f'(x)$$

$$(1-n) y = x n \frac{dy}{dx}$$

$$\int \frac{dy}{y} = \left(\frac{1-n}{n}\right)\int \frac{dx}{x}$$

$$\ln y = \ln x^{\left(\frac{1-n}{n}\right)} + C$$

$$\frac{y}{x^{\left(\frac{1-n}{n}\right)}} = C \Rightarrow y = C x^{\left(\frac{1-n}{n}\right)}$$

Type ① → Ⓑ :- If $\frac{dy}{dx} = f(ax + by + c)$

Then put $ax + by + c = t$ to convert into D.E of variable-separation type of t & x and finally resubstitute 't' as $ax + by + c$.

Q. 24]. Solve $\left(\sqrt{1 + x + y}\right)\left(\frac{dy}{dx}\right) = x + y - 1$

Ans

$$\frac{dy}{dx} = \frac{x + y - 1}{\sqrt{1 + x + y}}$$

→ M - I Put $x + y - 1 = t$

M - II -11- $x + y + 1 = t$

M - III -11- $x + y + 1 = t^2$

$\Rightarrow 1 + \frac{dy}{dx} = 2t\frac{dt}{dx}$.

$$2t\frac{dt}{dx} - 1 = \frac{t^2 - 2}{t}$$

$$\frac{dt}{dx} = \frac{t^2 + t - 2}{t^2} \Rightarrow \int \frac{t^2\,dt}{t^2 + t - 2} = \int dx \ldots \text{ Ans.}$$

(c) Homogeneous D.E^s :

$\frac{dy}{dx} = f(x,y)$ is called Homogeneous D.E if $x \to tx$ & $y \to ty$ and $\frac{dy}{dx}$ becomes independent of 't'.

Method to solve :- Put $y = vx$

i.e $\frac{dy}{dx} = v + x\frac{dv}{dx} \Rightarrow$ get var. sep in v & x

(अरे तो v, x और y से funcⁿ है ना कि constant, तो y, x से funcⁿ है i.e. when diff. $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$)

Q. (28) Solve $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$ & $y)_{x=0} = \sqrt{5}$

(A) $y^2 = 2x\sqrt{5} + 5$ (B) $y^2 = -2x\sqrt{5} + 5$

(C) $y^2 = 2x\sqrt{5} - 5$ (D) $y^2 = -2x\sqrt{5} - 5$

Ans. The given eqⁿ is like quadratic in $\frac{dy}{dx}$ यहाँ पे सीधे $\frac{dy}{dx} = f(..)$ नहीं दिया है। ई बात By Quadratic ~~rule~~ formula. $\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$

$$\frac{dy}{dx} = \frac{-2x \pm \sqrt{4x^2 + 4y^2}}{2y}$$

$$= \underbrace{\frac{-x + \sqrt{x^2+y^2}}{y} \quad \& \quad \frac{-x - \sqrt{x^2+y^2}}{y}}_{\text{These are homogenous eq}^n\text{ as checked by putting } x = tx \text{ \& } y = ty.}$$

$\Rightarrow$ $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \Rightarrow v + x\frac{dv}{dx} = \frac{-x + x\sqrt{1+v^2}}{xv}$ | $v + x\frac{dv}{dx} = \frac{-x - x\sqrt{1+v^2}}{vx}$

. . . . Ans.

Integration :- $\int \frac{2\ln y}{y} \cdot dy = (\ln y)^2 + C$

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