

Question

If $y' = [x - y] / [x + y]$, then its solution is _____.

Solution

Given $dy / dx = x - y / x + y$.

Put $y = vx$

$$dy / dx = v + x * [dv / dx]$$

$$v + x * [dv / dx] = [x - vx] / [x + vx]$$

$$v + x [dv / dx] = [1 - v] / [1 + v] \quad [1 + v] / [2 - (1 + v)^2] dv = dx / x$$

Integrating both sides, $\int [1 + v] / [2 - (1 + v)^2] dv = \int dx / x$

Put $(1 + v)^2 = t$

$$\Rightarrow 2(1 + v) dv = dt$$

$$[1 / 2] \int dt / [2 - t] = \int dx / x$$

$$[-1 / 2] \log(2 - t) = \log xc$$

$$[-1 / 2] \log[2 - (1 + v)^2] = \log xc$$

$$[-1 / 2] \log[-v^2 - 2v + 1] = \log xc$$

$$\log 1 / \sqrt{1 - 2v - v^2} = \log xc$$

$$x^2 c^2 (1 - 2v - v^2) = 1$$

$$y^2 + 2xy - x^2 = c_1.$$