

Question

$(x^2 + y^2) dy = xy dx$. If $y(x_0) = e$, $y(1) = 1$, then the value of $x_0 =$ _____.

Solution:

$$(x^2 + y^2) dy = xy dx$$

$$x(x dy - y dx) = -y^2 dy$$

$$[x * (y dx - x dy)] / y^2 = dy$$

$$[x / y] d(x / y) = dy / y$$

$$\text{Integrating, } x^2 / 2y^2 = \log_e y + c$$

$$\text{Given } y(1) = 1$$

$$c = 1 / 2$$

$$x^2 / 2y^2 = \log_e y + 1 / 2$$

$$\text{Now } y(x_0) = e$$

$$x_0^2 / 2e^2 - \log_e e - 1 / 2 = 0$$

$$x_0^2 = 3e^2$$

$$x_0 = \pm \sqrt{3}e$$