31. If $A = \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$, $B = \begin{bmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{bmatrix}$ and if A is invertible, then which of the following is not true?

a. $|A| = |B|$

b. $|A| = -|B|$

c. $|\text{adj } A| = |\text{adj } B|$

d. A is invertible if and only if B is invertible

31. a. $|B| = \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix}$ (Multiplying R_2 by -1)

$= -\begin{vmatrix} q & -b & y \\ p & -a & x \\ r & -c & z \end{vmatrix}$ (Multiplying C_2 by -1)

$= \begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix}$ (Changing R_1 with R_2)

$= -\begin{vmatrix} p & a & x \\ q & b & y \\ r & c & z \end{vmatrix}$

$= -\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$

Hence $|A| = -|B|$, obviously when $|A| \neq 0$, $|B| \neq 0$. Also, $|\text{adj } B| = |B|^2 = (-|A|)^2 = |A|^2$.