Q. 56 If $A=\frac{1}{\pi}\begin{bmatrix}\sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x)\end{bmatrix}$ and $B=\frac{1}{\pi}\begin{bmatrix}-\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x)\end{bmatrix}$,

then $A-B$ is equal to

(a) I (b) 0 (c) $2I$ (d) $\frac{1}{2}I$

Sol. *(d)* We have, $A=\begin{bmatrix}\frac{1}{\pi}\sin^{-1}x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi}\\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{1}{\pi}\cot^{-1}\pi x\end{bmatrix}$

and $B=\begin{bmatrix}\frac{-1}{\pi}\cos^{-1}x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi}\\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{-1}{\pi}\tan^{-1}\pi x\end{bmatrix}$

$$\therefore \quad A-B=\begin{bmatrix}\frac{1}{\pi}(\sin^{-1}x\pi+\cos^{-1}x\pi) & \frac{1}{\pi}\left(\tan^{-1}\frac{x}{\pi}-\tan^{-1}\frac{x}{\pi}\right)\\ \frac{1}{\pi}\left(\sin^{-1}\frac{x}{\pi}-\sin^{-1}\frac{x}{\pi}\right) & \frac{1}{\pi}\cot^{-1}\pi x+\tan^{-1}\pi x\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{\pi}\cdot\frac{\pi}{2} & 0\\ 0 & \frac{1}{\pi}\cdot\frac{\pi}{2}\end{bmatrix} \qquad \left[\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2} \text{ and } \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$=\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\frac{1}{2}I$$