

Q.22

$$\text{If } A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix} \text{ and}$$

$$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, \text{ then } 13(a^2 + b^2) \text{ is equal to}$$

25th Feb Morning Slot 2021

$$A = \begin{bmatrix} 0 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 0 \end{bmatrix}$$

$$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow I - A = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix} \{ \because |I - A| = \sec^2 \theta / 2 \}$$

$$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2 \frac{\theta}{2}} \begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix}$$

$$\Rightarrow (I + A)(I - A)^{-1}$$

$$= \frac{1}{\sec^2 \frac{\theta}{2}} \begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2 \frac{\theta}{2}} \begin{bmatrix} 1 - \tan^2 \frac{\theta}{2} & -2 \tan \frac{\theta}{2} \\ 2 \tan \frac{\theta}{2} & 1 - \tan^2 \frac{\theta}{2} \end{bmatrix}$$

$$a = \frac{1 - \tan^2 \frac{\theta}{2}}{\sec^2 \frac{\theta}{2}}$$

$$b = \frac{2 \tan \frac{\theta}{2}}{\sec^2 \frac{\theta}{2}}$$

$$\therefore a^2 + b^2 = 1$$

$$\Rightarrow 13(a^2 + b^2) = 13$$