

Q.11

Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3 . If $B = [b_{ij}]$, then b_{13} is equal to _____.

20th Jul Morning Shift 2021

Ans. 11

$$\text{Let } A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = I + C$$

$$\text{where, } I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$C^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$C^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = C^4 = C^5 = \dots\dots\dots$$

$$B = 7A^{20} - 20A^7 + 2I$$

$$= 7(I + C)^{20} + 20(I + C)^7 + 2I$$

$$= 7(I + 20C + {}^{20}C_2C^2) - 20(I + 7C + {}^7C_2C^2) + 2I$$

So

$$b_{13} = 7 \times {}^{20}C_2C^2 - 20 \times {}^7C_2 = 910$$