4. If $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$ then

a. Δ is independent of θ

b. Δ is independent of ϕ

c. Δ is a constant

d. $\left.\dfrac{d\Delta}{d\theta}\right]_{\theta=\pi/2} = 0$

4. b, d.

Applying $C_1 \to C_1 - (\cot\phi)\, C_2$, we get

$$\Delta = \begin{vmatrix} 0 & \sin\theta\sin\phi & \cos\theta \\ 0 & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta/\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$$

$= -\dfrac{\sin\theta}{\sin\phi}\,[-\sin\phi\sin^2\theta - \cos^2\theta\sin\phi]$ [expanding along C_1]

$= \sin\theta$

which is independent of ϕ. Also,

$$\frac{d\Delta}{d\theta} = \cos\theta \Rightarrow \left.\frac{d\Delta}{d\theta}\right]_{\theta=\pi/2} = \cos(\pi/2) = 0$$