

6. If A is an idempotent matrix satisfying, $(I - 0.4A)^{-1} = I - \alpha A$ where I is the unit matrix of the same order as that of A then the value of $|9\alpha|$ is equal to.

$$\text{Given } A^2 = A$$

$$\begin{aligned}\Rightarrow I &= (I - 0.4A)(I - \alpha A) \\ &= I - I\alpha A - 0.4AI + 0.4\alpha A^2 \\ &= I - A\alpha - 0.4A + 0.4\alpha A \\ &= I - A(0.4 + \alpha) + 0.4\alpha A\end{aligned}$$

$$\Rightarrow 0.4\alpha = 0.4 + \alpha$$

$$\Rightarrow \alpha = -2/3$$

$$\Rightarrow |9\alpha| = 6$$