

Q. 44 If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ and $A^{-1} = A'$, then find the value of α .

Sol. We have,

$$A = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

Also,

$$A^{-1} = A'$$

\Rightarrow

$$AA^{-1} = AA'$$

\Rightarrow

$$I = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

\Rightarrow

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$$

By using equality of matrices, we get

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

which is true for all real values of α .