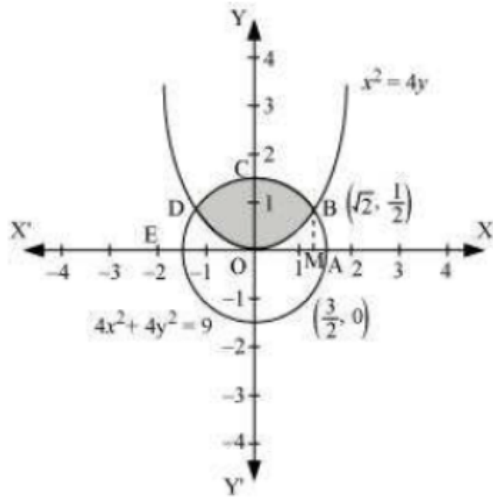


Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$

solution:

The required area is represented by the shaded area OBCDO.



Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the

point of intersection as $B \left(\sqrt{2}, \frac{1}{2} \right)$ and $D \left(-\sqrt{2}, \frac{1}{2} \right)$.

It can be observed that the required area is symmetrical about y -axis.

\therefore Area OBCDO = $2 \times$ Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are $(\sqrt{2}, 0)$.

Therefore, Area OBCO = Area OMBCO - Area OMBO

$$\begin{aligned} &= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}} dx \\ &= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\ &= \frac{1}{4} \left[x\sqrt{9-4x^2} + \frac{9}{2} \sin^{-1} \frac{2x}{3} \right]_0^{\sqrt{2}} - \frac{1}{4} \left[\frac{x^3}{3} \right]_0^{\sqrt{2}} \\ &= \frac{1}{4} \left[\sqrt{2}\sqrt{9-8} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} (\sqrt{2})^3 \\ &= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} \\ &= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} \\ &= \frac{1}{2} \left(\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right) \end{aligned}$$

Therefore, the required area OBCDO is

$$\left(2 \times \frac{1}{2} \left[\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \text{units}$$