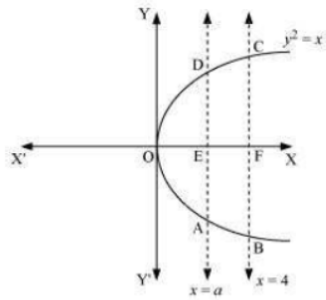


The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a .

Answer

The line, $x = a$, divides the area bounded by the parabola and $x = 4$ into two equal parts.

\therefore Area OAD = Area ABCD



It can be observed that the given area is symmetrical about x-axis.

\Rightarrow Area OED = Area EFCD

$$\begin{aligned}
 \text{Area OED} &= \int_0^a y \, dx \\
 &= \int_0^a \sqrt{x} \, dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\
 &= \frac{2}{3}(a)^{\frac{3}{2}} \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of EFCD} &= \int_0^4 \sqrt{x} \, dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\
 &= \frac{2}{3} \left[8 - a^{\frac{3}{2}} \right] \quad \dots(2)
 \end{aligned}$$

From (1) and (2), we obtain

$$\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3}\left[8 - (a)^{\frac{3}{2}}\right]$$

$$\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

Therefore, the value of a is $(4)^{\frac{2}{3}}$.