Question 2: If the system of linear equations

2x + 2ay + az = 0

2x + 3by + bz = 0 and

2x + 4cy + cz = 0,

where a, b, c ∈ R are non-zero and distinct; has non-zero solution, then

(a) a + b + c = 0

(b) 1/a, 1/b, 1/c are in A.P.

(c) a, b, c are in A.P.

(d) a, b, c are in G.P.

Answer: (b)

Solution:

Given system of linear equations

2x + 2ay + az = 0

2x + 3by + bz = 0 and

2x + 4cy + cz = 0,

Now,

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$R_2 \to R_2 - R_1 \; and \; R_3 \to R_3 - R_1$$

$$\begin{vmatrix} 2 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

=> (3b – 2a)(c – a) – (4c – 2a)(b – a) = 0

=> 3bc – 2ac – 3ab + $2a^2$ – [4bc – 4ac – 2ab + $2a^2$] = 0

=> –bc + 2ac – ab = 0

=> ab + bc = 2ac

=> 1/c + 1/a = 2/b

Which shows that 1/a, 1/b, 1/c are in A.P.