

Question 5: The system of linear equations

$$\lambda x + 2y + 2z = 5$$

$$2\lambda x + 3y + 5z = 8$$

$$4x + \lambda y + 6z = 10 \text{ has:}$$

- (a) no solution when $\lambda = 2$
- (b) infinitely many solutions when $\lambda = 2$
- (c) no solution when $\lambda = 8$
- (d) a unique solution when $\lambda = -8$

Answer: (a)

Solution:

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$$

$$\Rightarrow D = -\lambda^2 - 6\lambda + 16$$

Now, $D = 0$

$$\Rightarrow \lambda^2 + 6\lambda - 16 = 0$$

$$\Rightarrow \lambda = -8 \text{ or } 2$$

For $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix} = 40 + 4 - 28 \neq 0$$

Therefore, Equations have no solution for $\lambda = 2$.