11. The system of equations

$\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$,

$x + y + \alpha z = \alpha - 1$ has no solution, if α is (2005)

1) -2 2) either -2 or 1

3) not -2 4) 1

Ans.

(1) $\alpha x + y + z = \alpha - 1$ has no solution

$x + \alpha y + z = \alpha - 1$, $x + y + z\alpha = \alpha - 1$

$$\Rightarrow \Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$$

$$\Rightarrow \alpha(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1) = 0$$

$$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 1 - 1] = 0$$

$$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 2] = 0$$

$$\Rightarrow [\alpha^2 + 2\alpha - \alpha - 2] = 0$$

$$\Rightarrow (\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)] = 0$$

$$\Rightarrow (\alpha - 1) = 0, \alpha + 2 = 0 \Rightarrow \alpha = -2, 1 : \text{ but } \alpha \neq 1.$$