$$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

and $Q = PAP^T$, then $P(Q^{2005})P^T$ equal to _____.

Solution:

If $Q = PAP^T$, $P^T Q = AP^T$,

(as $PP^T = I$) $P^T Q^{2005} P = A P^T Q^{2004} P$

$= A^2 P^T Q^{2003} P$

$= A^3 P^T Q^{2002} P$

$= A^{2004} P^T (QP)$

$= A^{2004} P^T (PA)$ ($Q = PAP^T \Rightarrow QP = PA$)

$= A^{2005}$

$A^{2005} =$

$$\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$