19. Let a, b, c any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x=cy+bz$, $y=az+cx$, $z=bx+ay$. Then $a^2+b^2+c^2+2abc$ is equal to (2008)

1) 2 2) –1 3) 0 4) 1

Ans.

(4) The system of equations $x-cy-bz=0$, $cx-y+az=0$ and $bx+ay-z=0$ have non-trivial solution if $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}=0$

$\Rightarrow 1(1-a^2) + c(-c-ab)-b(ca+b)=0$

$\Rightarrow a^2+b^2+c^2+2abc=1$