If PQR be a triangle of area Δ with $a = 2$, $b = 7/2$ and $c = 5/2$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R, respectively. Then $(2 \sin P - \sin 2P)/(2 \sin P + \sin 2P)$ is equal to

a) $3/4\Delta$

(b) $45/4\Delta$

(c) $(3/4\Delta)2$

(d) $(45/4\Delta)5$

solution

Given $a = 2$, $b = 7/2$ and $c = 5/2$

$s = (a + b + c)/2$

$= 4$

$(2 \sin P - \sin 2P)/(2 \sin P + \sin 2P) = (2 \sin P - 2 \sin P \cos P)/(2 \sin P + 2 \sin P \cos P)$

$= 2 \sin P(1 - \cos P)/2 \sin P(1 + \cos P)$

$= (1 - \cos P)/(1 + \cos$

P)

$= 2 \sin^2 (P/2)/2 \cos^2 (P/2)$

$= \tan^2 (P/2)$

$= (s - b)(s - c)/s(s - a)$ (since $\tan (P/2) = \sqrt{((s - b)(s - c)/s(s - a))}$

$= (s - b)^2(s - c)^2/s(s - a)(s - b)(s - c)$ (multiply numerator and denominator by $(s - b)(s - c)$)

$= (4 - 7/2)^2(4 - 5/2)^2/\Delta^2$

solution

$= (½ \times 3/2)^2/\Delta^2$

$= (3/4\Delta)^2$