If in a triangle ABC, $\cos A \cos B + \sin A \sin B \sin C = 1$, show that a b : c = 1 :1 2.

Solution:

Given that $\cos A \cos B + \sin A \sin B \sin C = 1$

Now, this gives $\sin C = (1 - \cos A \cos B)/\sin A \sin B$ ... (1)

Also, we know $\sin C \leq 1$, which implies that

$(1 - \cos A \cos B)/\sin A \sin B \leq 1$

Hence, $(1 - \cos A \cos B) \leq \sin A \sin B$

$\Rightarrow 1 \leq \sin A \sin B + \cos A \cos B$

$\Rightarrow 1 \leq \cos (A-B)$

$\Rightarrow \cos (A-B) \geq 1$

But, $\cos \theta \leq 1$, so we have

$\Rightarrow \cos (A-B) = 1$.

$\Rightarrow A-B =$

0.

Putting $A = B$ in equation (1) we have,

$\sin C = (1-\cos 2A)/\sin 2A$

$\Rightarrow \sin C = 1 \Rightarrow C = \pi/2$

Now, $A + B + C = \pi$

$\Rightarrow A + B = \pi/2$

Since $A = B$ and $C = \pi/2$, so we have $A = \pi/4$

$\sin A \quad \sin B \quad \sin C = \sin \pi/4 \quad \sin \pi/4 \quad \sin \pi/4$

$= 1/\sqrt{2} : 1/\sqrt{2} : 1$

Hence, $a \quad b \quad c = 1 : 1 : \sqrt{2}$.