

Sine rule

This rule states that the sides of any triangle are proportional to the sines of the angle opposite to them in $\triangle ABC$, i.e.

$$a/\sin A = b/\sin B = c/\sin C$$

Note:

The rule can further be expressed as $(\sin A)/a = (\sin B)/b = (\sin C)/c$

The rule is extremely useful for expressing the sides of a triangle in terms of sines of angle or vice versa as per the requirement of the question. Hence, for the form $a/\sin A = b/\sin B = c/\sin C = k$, we have,

$$a = k \sin A, b = k \sin B, c = k \sin C$$

and for $(\sin A)/a = (\sin B)/b = (\sin C)/c = l$, we have

$$\sin A = la, \sin B = lb, \sin C = lc$$

Cosine rule

In any triangle ABC , the following results hold good:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = (a^2 + c^2 - b^2)/2ac$$

$$\cos C = (a^2 + b^2 - c^2)/2ab$$

Note: In case

$$\angle A = 60^\circ, \text{ then } b^2 + c^2 - a^2 = bc$$

$$\angle B = 60^\circ, \text{ then } a^2 + c^2 - b^2 = ac$$

$$\angle C = 60^\circ, \text{ then } a^2 + b^2 - c^2 = ab$$

Trigonometric ratios of half-angles:

$$\sin A/2 = \sqrt{(s-b)(s-c)/bc}$$

$$\sin B/2 = \sqrt{(s-b)(s-c)/bc}$$

$$\sin C/2 = \sqrt{(s-a)(s-b)/ab}$$

$$\cos A/2 = \sqrt{s(s-a)/bc}$$

$$\cos B/2 = \sqrt{s(s -$$

$b)/ca$

$$\cos C/2 = \sqrt{s(s-c)/ab}$$

$$\tan A/2 = \sqrt{(s-b)(s-c)/s(s-a)}$$

$$\tan B/2 = \sqrt{(s-c)(s-a)/s(s-b)}$$

$$\tan C/2 = \sqrt{(s-a)(s-b)/s(s-c)}$$

Projection rule:

$$a = b \cos C + c \cos B$$

$$b = c \cos A + a \cos C$$

$$c = a \cos B + b \cos A$$

Semi-perimeter of the triangle

If 's' is assumed to be the perimeter of the triangle then $s = \frac{a + b + c}{2}$

Area of a triangle

If Δ is the area of the triangle ABC then

$$\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C$$