

The radii r_1, r_2, r_3 of escribed circle of a triangle ABC are in H.P. If its area is 24 cm^2 and its perimeter is 24 cm , find the length of its sides.

Solution:

Let a, b, c be the length of the sides of the triangle, s be the semi perimeter and Δ be the area.

$$r_1 = \Delta / (s - a) = 24 / (12 - a)$$

$$r_2 = \Delta / (12 - b)$$

$$r_3 = \Delta / (s - c) = 24 / (12 - c)$$

Since r_1, r_2, r_3 are in H.P.

$\Rightarrow 1/r_1, 1/r_2, 1/r_3$ are in A.P.

$$\Rightarrow 2/r_2, 1/r_1, 1/r_3 \text{ or } 2(12 - b) / 24 = 12 - a / 24 + 12 - c / 24$$

$$\text{or } 2b = a + c \dots\dots (1)$$

$$\text{Also, } a + b + c = 24 \dots\dots (2)$$

From (1) and (2), $b = 9 \text{ cm}$ and $a + c = 16$

cm

$$\text{Now, } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow 24 = \sqrt{12(12-a)(12-b)(12-c)}$$

$$\Rightarrow 24 \times 24 = 12(12-a) \times 4 \times (12-16+a)$$

$$\Rightarrow 12 = (12-a)(a-4)$$

$$\Rightarrow a^2 - 16a + 60 = 0$$

$$\Rightarrow a = 6 \text{ or } 10.$$

When $a = 6$, $c = 10$ and when $a = 10$, $c = 6$.

Hence sides are 6, 8, 10 or 10, 8, 6.