If in a triangle ABC, $\sin C + \cos C + \sin(2B + C) - \cos(2B + C) = 2\sqrt{2}$, the the triangle ABC is

(a) equilateral

(b) scalene

(c) isosceles right-angled

(d) obtuse-angled

Solution

Given $\sin C + \cos C + \sin(2B + C) - \cos(2B + C) = 2\sqrt{2}$

Rearranging the equation, we get

$\sin C + \sin(2B + C) + \cos C - \cos(2B + C) = 2\sqrt{2}$ (i)

We know that $\sin A + \sin B = 2\sin(A + B)/2 \cos(A - B)/2$

So (i) => $2\sin(B + C)\cos B + 2\sin(B + C)\sin B = 2\sqrt{2}$

=> $\sin(\pi - A)\cos B + \sin(\pi - A)\sin B = \sqrt{2}$

=> $\sin A \cos B + \sin A \sin B =$

$\sqrt{2}$

=> $\sin A(\cos B + \sin B) = \sqrt{2}$

Divide both sides by $\sqrt{2}$

$\sin A((1/\sqrt{2}) \cos B + (1/\sqrt{2}) \sin B) = 1$

We know $\sin \pi/4 = \cos \pi/4 = 1/\sqrt{2}$

=> $\sin A(\sin \pi/4 \cos B + \sin \pi/4 \sin B) = 1$

Solution

=> $\sin A \sin (B + \pi/4) = 1$

=> $\sin A = 1$ and $\sin (B + \pi/4) = 1$

=> $A = 900$, and $(B + \pi/4) = \pi/2$

$B = \pi/4 = 450$

So $C = 450$

Thus the triangle is isosceles right angled.

Hence option (c) is the answer.