

If d_1, d_2, d_3 are the diameters of the three escribed circles of a triangle, then $d_1d_2 + d_2d_3 + d_3d_1$ is equal to

(a) $4s^2$

(b) Δ^2

(c) $4\Delta^2$

(d) $2\Delta^2$

Solution

We know $d_1 = 2r_1, d_2 = 2r_2, d_3 = 2r_3$

We have $r_1 = \Delta/(s - a)$

$$r_2 = \Delta/(s - b)$$

$$r_3 = \Delta/(s - c)$$

$$d_1d_2 + d_2d_3 + d_3d_1 = 4(r_1r_2 + r_2r_3 + r_3r_1)$$

$$= 4\Delta^2(1/(s - a)(s - b) + 1/(s - b)(s - c) + (s - c)(s - a))$$

$$= 4\Delta^2(s - c + s - a + s - b)/(s - a)(s - b)(s -$$

c)

$$= 4\Delta^2 (s - (a + b + c)) / (s - a)(s - b)(s - c)$$

$$= 4\Delta^2 (s - 2s) / (s - a)(s - b)(s - c)$$

$$= 4\Delta^2 s / (s - a)(s - b)(s - c)$$

Multiply numerator and denominator by s

$$= 4\Delta^2 s^2 / s(s - a)(s - b)(s - c)$$

Solution

$$= 4\Delta^2 s^2 / \Delta^2$$

$$= 4s^2$$