

Let ABC and ABC' be two non congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30^\circ$. The absolute value of the difference between the areas of these triangles is

Solution

Given $AB = 4$, $AC = AC' = 2\sqrt{2}$

$$\angle B = 30^\circ$$

Using cosine rule, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\frac{\sqrt{3}}{2} = \frac{a^2 + 16 - 8}{8a}$$

$$\frac{\sqrt{3}}{2} = \frac{a^2 + 8}{8a}$$

$$a^2 - 4\sqrt{3}a + 8 = 0$$

Let a_1 and a_2 be the roots of this equation.

$$a_1 + a_2 = 4\sqrt{3}$$

$$a_1 a_2 = 8$$

$$\frac{|a_1 - a_2|}{2} = \frac{(a_1 + a_2)^2 -$$

$$4a_1a_2$$

$$= 48 - 32$$

$$= 16$$

Solution

$$|a_1 - a_2| = 4$$

$$|\Delta_1 - \Delta_2| = \frac{1}{2} |a_1 - a_2| c \sin B$$

$$= \frac{1}{2} 4 \sin 30^\circ \times 4$$

$$= 4$$