Que 15:

Evalute $\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)} dx.$

[1993 - 5 Marks]

solution:

$$I = \int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)} dx = \int_2^3 \frac{2x^3(x^2-1) + (x^2+1)^2}{(x^2+1)^2 (x^2-1)} dx$$

$$= \int_2^3 \frac{2x^3}{(x^2+1)^2} + \int_2^3 \frac{1}{x^2-1} dx \ = \int_2^3 \frac{x^2.2x}{(x^2+1)^2} + \left[\frac{1}{2}\log\frac{x-1}{x+1}\right]_2^3$$

Put $x^2 + 1 = t,\ 2x\,dx = dt$

when $x \to 2, t \to 5\ ,\ x \to 3, t \to 10$

$$\therefore I = \int_5^{10} \frac{t-1}{t^2} dt + \frac{1}{2}\left(\log\frac{2}{4} - \log\frac{1}{3}\right)$$

$$= \int_5^{10} \left(\frac{1}{t} - \frac{1}{t^2}\right) dt + \frac{1}{2}\log\frac{3}{2} \ = \left(\log|t| + \frac{1}{t}\right)_5^{10} + \frac{1}{2}\log\frac{3}{2}$$

$$= \log 10 - \log 5 + \frac{1}{10} - \frac{1}{5} + \frac{1}{2}\log\frac{3}{2}$$

$$= \frac{1}{2}\left[2\log 2 + \log\frac{3}{2}\right] - \frac{1}{10}$$

$$= \frac{1}{2}\log 6 - \frac{1}{10}$$