Que 9:

Sketch the region bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ and find its area.

[1985 - 5 Marks]

solution:

The given curves are

$$y=\sqrt{5-x^2} \qquad \ldots \text{(i)}$$

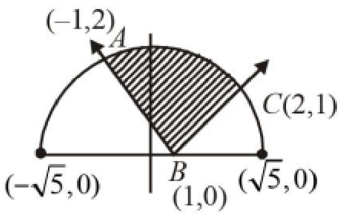
$$y=|x-1| \qquad \ldots \text{(ii)}$$

We can clearly see that (on squaring both sides of (1)) eq. (i) represents a circle. But as y is + ve sq. root, ∴ (1) represents upper half of circle with centre (0, 0) and radius $\sqrt{5}$.

Eq. (ii) represents the curve

$$y=\begin{cases}-x+1 & \text{if } x<1\\ x-1 & \text{if } x\geq 1\end{cases}$$

Graph of these curves are as shown in figure with point of intersection of $y=\sqrt{5-x^2}$ and $y=-x+1$ as $A(-1,2)$ and of $y=\sqrt{5-x^2}$ and $y=x-1$ as $C(2,1)$.



The required area = Shaded area

$$=\int_{-1}^{2}(y_{(1)}-y_{(2)})dx=\int_{-1}^{2}\sqrt{5-x^2}\,dx-\int_{-1}^{2}|x-1|\,dx$$

$$=\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_{-1}^{2}-\int_{1}^{1}\{-(x-1)\}\,dx-\int_{1}^{2}(x-1)dx$$

$$= \left(\frac{2}{2}\sqrt{5-4} + \frac{5}{4}\sin^{-1}\frac{2}{\sqrt{5}} \right) - \left(\frac{-1}{2}\sqrt{5-1} + \frac{5}{2}\sin{-1}\left(\frac{-1}{\sqrt{5}} \right) \right)$$

$$- \left(\frac{-x^2}{2} + x \right)_{-1}^{1} - \left(\frac{x^2}{2} - x \right)_{1}^{2}$$

$$= 2 + \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}} \right] - 2 - \frac{1}{2}$$

$$= \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}\frac{2}{\sqrt{5}} \right] - \frac{1}{2} = \frac{5}{2}\left(\frac{\pi}{2} \right) - \frac{1}{2}$$

$= \dfrac{5\pi - 2}{4}$ square units.