Que 8:

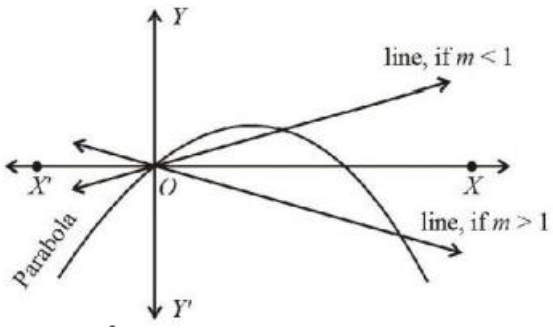
For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2?

[1999 - 3 Marks]

(a) -4 (b) -2 (c) 2 (d) 4

solution:

(b, d)



The given curve is $y = x - x^2$ and $y = mx$

The two curves meet at

$mx = x - x^2$ or $x^2 = x(1-m)$, $\therefore \; x = 0, 1-m$

The region bounded by curves

$$= \int_P^L (y_1 - y_2)dx = \int (x - x^2 - mx)dx$$

Clearly m < 1 or m > 1, but m ≠ 1

Now, $\int_0^{1-m} (1 - x - x^2)dx = \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3} \right]_0^{1-m}$

$$= \frac{9}{2}, \text{if } m<1$$

or $(1-m)^3 = 27 \Rightarrow 1 - m = -3$, $\therefore \; m = -2$

But if m >1 then 1– m is – ive, then $\int_{1-m}^{0} (1 - x - x^2)dx$

$$= \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3} \right]_{1-m}^{0} = \frac{9}{2}$$

$\therefore \; (1-m)^3 = -27 \Rightarrow 1 - m = -3$, or $1 - m = -3$, $\therefore$ m = 4.