

Que 1:

Find the area of the region bounded by the curve $C : y = \tan x$, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis.

[1988 - 5 Marks]

solution:

The given curve is $y = \tan x$... (i)

Let A be the point on (i) where $x = \pi/4$

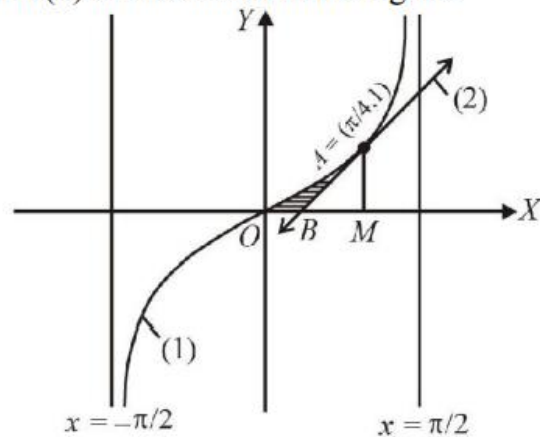
$$\therefore y = \tan \pi/4 = 1$$

So, co-ordinates of A are $(\pi/4, 1)$

$$\therefore \text{Equation of tangent at } A \text{ is } y - 1 = 2(x - \pi/4)$$

$$\text{or } y = 2x + 1 - \pi/2 \quad \dots \text{(ii)}$$

The graph of (1) and (2) are as shown in the figure.



Tangent (2) meets x-axis at, $L\left(\frac{\pi-2}{4}, 0\right)$

Now the required area = shaded area
= Area $OAMO$ - Ar(ΔABM)

$$\begin{aligned} &= \int_0^{\pi/4} \tan x \, dx - \frac{1}{2}(OM - OB)AM \\ &= [\log \sec x]_0^{\pi/4} - \frac{1}{2} \left\{ \frac{\pi}{4} - \frac{\pi-2}{4} \right\} \cdot 1 = \frac{1}{2} \left[\log 2 - \frac{1}{2} \right] \text{ sq. units.} \end{aligned}$$