In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$?

solutions:

The given equations of parabola are

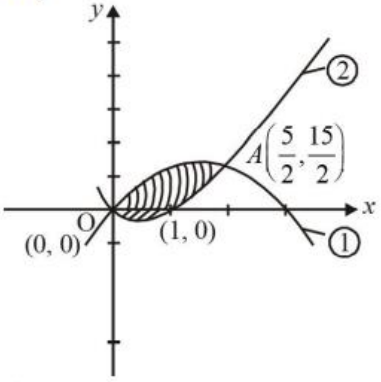
$y = 4x - x^2$ or $(x-2)^2 = -(y-4)$ ..... (i)

and $y = x^2 - x$ or $\left(x - \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$ ..... (ii)

Solving the equations of two parabolas we get their points of intersection as

$$O(0,0), A\left(\frac{5}{2}, \frac{15}{4}\right)$$

Here the area below x-axis,



$$A_1 = \int_0^1 (-y_2)\,dx = \int_0^1 (x - x^2)\,dx$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{6} \text{ sq. units.}$$

Area above x-axis,

$$A_2 = \int_0^{5/2} (4x - x^2)\,dx - \int_1^{5/2} (x^2 - x)\,dx$$

$$= \left(2x^2 - \frac{x^3}{3}\right)_0^{5/2} - \left(\frac{x^3}{3} - \frac{x^2}{2}\right)_1^{5/2}$$

$$= \left(\frac{25}{2} - \frac{125}{24}\right) - \left[\left(\frac{125}{24} - \frac{25}{8}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right] = \frac{121}{24}$$

$\therefore$ Ratio of areas above x- axis and below x – axis.

$$A_2 : A_1 = \frac{121}{24} : \frac{1}{6} = \frac{121}{4} = 121 : 4$$