The area of the region between the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x=0$ and $x=\frac{\pi}{4}$ is

[2008]

(a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

solution:

(b) Area $= \int_0^{\pi/4} \left[\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right]$

$$\int^{/4} \left(\sqrt{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}} \quad \sqrt{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}}\right) dx$$

$$= \int_0^{\pi/4} \frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}\,dx$$

Let $\tan\frac{x}{2} = \text{t} \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}\,dx = dt \Rightarrow dx = \frac{2}{1+t^2}\,dt$

When $x = 0$, then t = 0 and when $x=\frac{\pi}{4}$, then $t=\tan\frac{\pi}{8}$

$$\therefore A = \int_0^{\tan\frac{\pi}{8}} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt = \int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$$