Que 10:

The area enclosed by the curves $y = \sin x + \cos x$ and

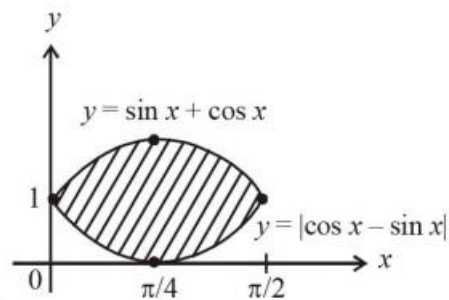$y = |\cos x - \sin x|$ over the interval $\left[0, \dfrac{\pi}{2}\right]$ is

(a) $4(\sqrt{2} - 1)$

(b) $2\sqrt{2}(\sqrt{2} - 1)$

(c) $2(\sqrt{2} + 1)$

(d) $2\sqrt{2}(\sqrt{2} + 1)$

solution:

(b) The rough graph of $y = \sin x + \cos x$ and

$y = |\cos x - \sin x|$ suggest the required area is

$= \displaystyle\int_0^{\pi/2} [(\sin x + \cos x) - |\cos x - \sin x|]\, dx$

$= \displaystyle\int_0^{\pi/4} 2\sin x\, dx + \int_{\pi/4}^{\pi/2} 2\cos x\, dx$

$= 2\left[(-\cos x)_0^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2}\right] = 2\sqrt{2}(\sqrt{2} - 1)$