Que 9:

The area of the region described by

$A=\{(x,y): x^2+y^2 \le 1 \text{ and } y^2 \le 1-x\}$ is:

[Main 2014]

(a) $\frac{\pi}{2}-\frac{2}{3}$

(b) $\frac{}{2}\ \frac{}{3}$

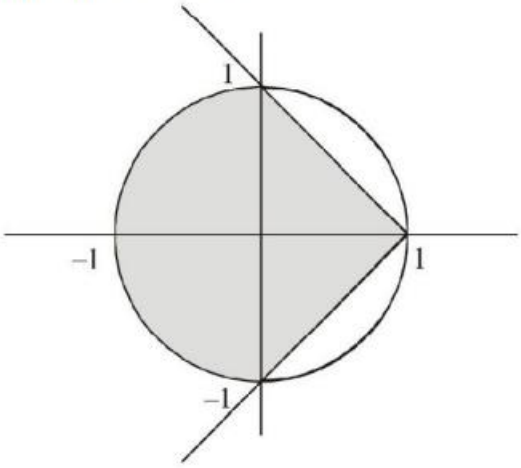
(c) $\frac{\pi}{2}+\frac{4}{3}$

(d) $\frac{\pi}{2}-\frac{4}{3}$

solutions:

(c) Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.

Intersecting points are $x = 0, 1$



Area of shaded portion is the required area.

So, Required Area = Area of semi-circle

+ Area bounded by parabola

$$= \frac{\pi r^2}{2} + 2\int_0^1 \sqrt{1-x}\,dx = \frac{\pi}{2} + 2\int_0^1 \sqrt{1-x}\,dx$$

($\because$ radius of circle = 1)

$$= \frac{\pi}{2} + 2\left[\frac{(1-x)^{3/2}}{-3/2}\right]_0^1 = \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3} \text{ sq. unit}$$