

Que 7:

The area of the region $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ is

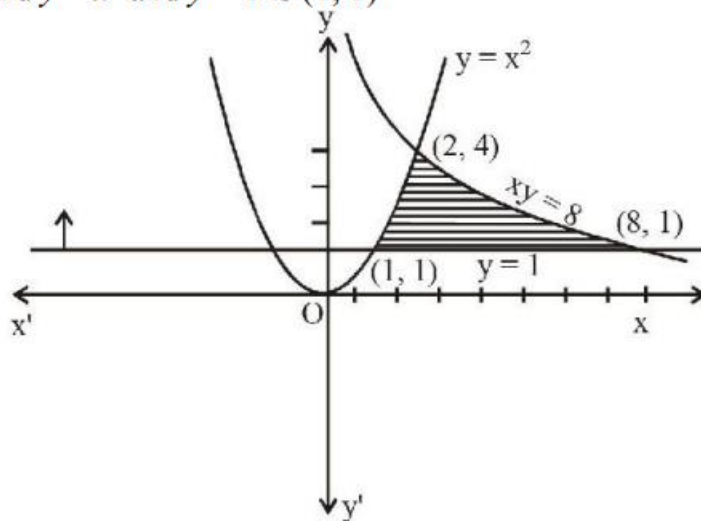
[Adv. 2018]

- (a) $8 \log_e 2 - \frac{14}{3}$
- (b) $16 \log_e 2 - \frac{14}{3}$
- (c) $8 \log_e 2 - \frac{7}{3}$
- (d) $16 \log_e 2 - 6$

solutions:

(b) $xy \leq 8, 1 \leq y \leq x^2$

Intersection points of $xy = 8$ and $y = 1$ is $(8, 1)$; $xy = 8$ and $y = x^2$ is $(2, 4)$ and $y = x^2$ and $y = 1$ is $(1, 1)$



$$\begin{aligned} \text{Required area} &= \int_1^2 x^2 dx + \int_2^8 \frac{8}{x} dx - \int_1^8 1 dx \\ &= \left[\frac{x^3}{3} \right]_1^2 + [8 \ln x]_2^8 - [x]_1^8 \\ &= \frac{8}{3} - \frac{1}{3} + 8 \ln 8 - 8 \ln 2 - (8 - 1) \\ &= \frac{7}{3} + 24 \ln 2 - 8 \ln 2 - 7 = 16 \ln 2 - \frac{14}{3} \end{aligned}$$