8. Let the functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=e^{x-1}-e^{-|x-1|}$ and $g(x)=\frac{1}{2}(e^{x-1}+e^{1-x})$.

Then the area of the region in the first quadrant bounded by the curves $y=f(x)$, $y=g(x)$ and $x=0$ is

[Adv. 2020]

(a) $(2-\sqrt{3})+\frac{1}{2}(e-e^{-1})$

(b) $(2+\sqrt{3})+\frac{1}{2}(e-e^{-1})$

(c) $(2-\sqrt{3})+\frac{1}{2}(e+e^{-1})$

(d) $(2+\sqrt{3})+\frac{1}{2}(e+e^{-1})$

solutions:

(a) □ $f(x)=e^{x-1}-e^{|x-1|}$

$$\therefore f(x)=\begin{cases} 0 & x\le 1 \\ e^{x-1}-e^{1-x} & x\ge 1 \end{cases}$$

and $g(x)=\frac{1}{2}\left(e^{x-1}+e^{1-x}\right)$

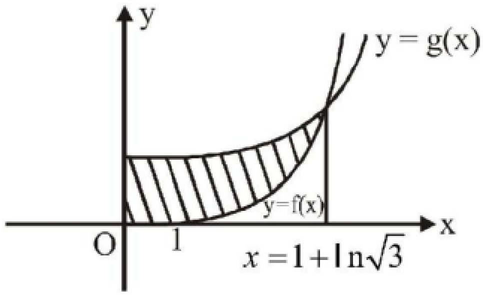
If $f(x)=g(x)$

$$\Rightarrow \quad e^{x-1}-e^{-(x-1)}=\frac{e^{x-1}+e^{1-x}}{2}$$

$$\Rightarrow \quad e^{2(x-1)}=3$$

$$\Rightarrow \quad x=\frac{1}{2}\ell n3+1$$

$$\Rightarrow \quad x=1+\ell n\sqrt{3}$$



So bunded area $=\int_0^{\frac{1}{2}\ell n3+1} g(x)\,dx-\int_1^{\frac{1}{2}\ell n3+1} f(x)\,dx$

$$=\frac{1}{2}\left[e^{x-1}-e^{1-x}\right]_0^{\frac{1}{2}\ell n3+1}-\left[e^{x-1}+e^{1-x}\right]_1^{\frac{1}{2}\ell n3+1}$$

$$=2-\sqrt{3}+\frac{1}{2}\left(e-\frac{1}{e}\right)$$