3. Given: $f(x) = \begin{cases} x &, \ 0 \le x < \dfrac{1}{2} \\ \dfrac{1}{2} &, \ x = \dfrac{1}{2} \\ 1-x &, \ \dfrac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :

[Main Jan. 9, 2020 (II)]

(a) $\frac{1}{3} + \frac{\sqrt{3}}{4}$

(b) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

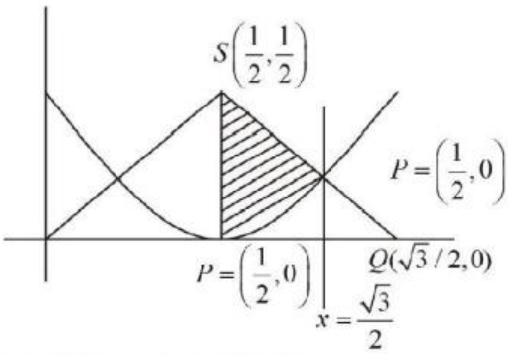
(c) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

(d) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

solution:

3. **(b)** Coordinates of $P\left(\frac{1}{2}, 0\right), Q\left(\frac{\sqrt{3}}{2}, 0\right), R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$

and $S\left(\frac{1}{2}, \frac{1}{2}\right)$



Required area = Area of trapezium $PQRS$

$$- \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\left(x - \frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$