

Que 1:

Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$.

[1981 - 4 Marks]

Solution:

To find the area bounded by the curves

$$x^2 = 4y \quad \dots(i)$$

$$\text{and } x - 4y = -2 \Rightarrow \frac{x}{-2} + \frac{y}{1/2} = 1 \quad \dots(ii)$$

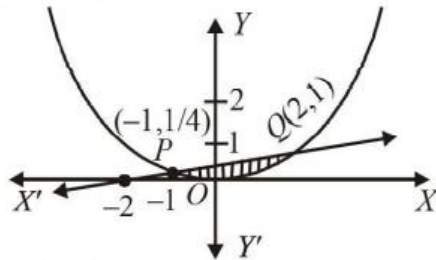
The intercepts of straight line (2) are -2 and $1/2$ with axes.

From (i) and (ii),

$$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = 2, -1 \Rightarrow y = 1, 1/4$$

$\therefore P(-1, 1/4)$ and $Q(2, 1)$.



Shaded region in the fig is the req area.

$$\therefore \text{Required area} = \int_{-1}^2 (y_{\text{line}} - y_{\text{parabola}}) dx$$

$$= \int_{-1}^2 \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx = \frac{1}{4} \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2$$

$$= 9/8 \text{ sq. units}$$