Q1. If $y + \dfrac{d}{dx}(xy) = x(\sin x + \log x)$, then find solution of this differential eqn.

Solution: The given differential equation can be written as:

$$x\dfrac{dy}{dx} + 2y = x(\sin x + \log x)$$

$$\dfrac{dy}{dx} + \dfrac{2}{x} \cdot y = (\sin x + \log x) \qquad \text{This is linear differential form.}$$

$$\boxed{e^{\int \frac{2}{x}dx} = e^{\log x^2} = x^2}$$

$$\therefore \quad y \cdot x^2 = \int x^2 (\sin x + \log x)\,dx + c$$

$$y \cdot x^2 = -x^2\cos x + 2x\sin x + 2\cos x + \dfrac{x^3}{3}\log x - \dfrac{x^3}{9} + c$$

$$\Rightarrow \quad \boxed{y = -\cos x + \dfrac{2\sin x}{x} + \dfrac{2}{x^2}\cos x + \dfrac{x}{3}\log x - \dfrac{x}{9} + \dfrac{c}{x^2}}$$