Also, $\sum_{i=1}^{n_1}(x_i-\bar{x})^2 = n_1 s_1^2 = n_1(\bar{x}_1-\bar{x})^2 = n_1 s_1^2 + n_1 d_1^2$

Where, $d_1 = (\bar{x}_1 - \bar{x})$

Similarly, $\sum_{j=1}^{n_2}(y_j-\bar{x})^2 = \sum_{j=1}^{n_2}(y_j-\bar{x}_i+\bar{x}_i-\bar{x})^2 = n_2 s_2^2 + n_2 d_2^2$

where, $d_2 = \bar{x}_2 - \bar{x}$

Combined SD, $\sigma = \sqrt{\frac{[n_1(s_1^2+d_1^2)+n_2(s_2^2+d_2^2)]}{n_1+n_2}}$

where, $d_1 = \bar{x}_1 - \bar{x} = \bar{x}_1 - \left(\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}\right) = \frac{n_2(\bar{x}_1-\bar{x}_2)}{n_1+n_2}$

and $d_2 = \bar{x}_2 - \bar{x} = \bar{x}_2 - \frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2} = \frac{n_1(\bar{x}_2-\bar{x}_1)}{n_1+n_2}$

$\therefore \qquad \sigma^2 = \frac{1}{n_1+n_2}\left[n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2(\bar{x}_1+\bar{x}_2)^2}{(n_1+n_2)^2} + \frac{n_2 n_1(\bar{x}_2-\bar{x}_1)^2}{(n_1+n_2)^2}\right]$

Also, $\sigma = \sqrt{\frac{n_1 s_2^2 + n_2 s_2^2}{n_1+n_2} + \frac{n_1 n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}}$