

3 JEE Main 2015 (Offline)

MCQ (Single Correct Answer)

Let $y(x)$ be the solution of the differential equation

$(x \log x) \frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then $y(e)$ is equal to :

A 2

B $2e$

C e

D 0

Explanation

$$\text{Given, } \frac{dy}{dx} + \left(\frac{1}{x \log x} \right) y = 2$$

$$I. F. = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$$

$$y \cdot \log x = \int 2 \log x dx + c$$

$$y \log x = 2 [x \log x - x] + c$$

$$\text{Put } x = 1, y \cdot 0 = -2 + c \Rightarrow c = 2$$

$$\text{Put } x = e$$

$$y \log e = 2e (\log e - 1) + c \Rightarrow y(e) = c = 2$$

Q. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = (2 + 5y)(5y - 2)$, then the value of

$\lim_{x \rightarrow -\infty} f(x)$ is

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$$\frac{dy}{dx} = 25y^2 - 4$$

$$\text{So, } \frac{dy}{25y^2 - 4} = dx$$

Sol. 0.4 Integrating, $\frac{1}{25} \times \frac{1}{2 \times \frac{2}{5}} \ln \left| \frac{y - \frac{2}{5}}{y + \frac{2}{5}} \right| = x + c$

$$\Rightarrow \ln \left| \frac{5y - 2}{5y + 2} \right| = 20(x + c)$$

Now, $c = 0$ as $f(0) = 0$

$$\text{Hence } \left| \frac{5y - 2}{5y + 2} \right| = e^{(20x)}$$

$$\lim_{x \rightarrow -\infty} \left| \frac{5f(x) - 2}{5f(x) + 2} \right| = \lim_{x \rightarrow -\infty} e^{(20x)}$$

$$\text{Now, RHS} = 0 \Rightarrow \lim_{x \rightarrow -\infty} (5f(x) - 2) = 0$$

$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \frac{2}{5}$$