A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is -

A $\frac{1}{6}\sqrt{\frac{7}{3}}$

B $\frac{\sqrt{5}}{6}$

C $\frac{1}{2}$

D $\frac{1}{3}\sqrt{\frac{7}{3}}$

$y - x^{3/2} = 7 \ (x \geq 0)$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$\left(\frac{3}{2} \sqrt{x} \right) \left(\frac{7-y}{\frac{1}{2} - x} \right) = -1$

$\left(\frac{3}{2} \sqrt{x} \right) \left(\frac{-x^{3/2}}{\frac{1}{2} - x} \right) = -1$

$\frac{3}{2} \cdot x^2 = \frac{1}{2} - x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x+1)(3x-1) = 0$

$\therefore x = -1 \ \text{(rejected)}$

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3} \right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3} \right)^2 + \left(\frac{1}{3} \right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{3+4}{9 \times 12}}$