30. Find the equation of the curve through the point (1, 0) if the slope of the tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$.

Sol. It is given that, slope of tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$.

$$\therefore \quad \frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \quad \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \frac{dx}{x(x+1)}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\Rightarrow \quad \log(y-1) = \log x - \log(x+1) + \log C$$

$$\Rightarrow \quad \log(y-1) = \log \frac{Cx}{x+1}$$

$$\Rightarrow \quad y-1 = \frac{Cx}{x+1} \qquad \text{(i)}$$

Since curve passes through point (1, 0)

$$\therefore \quad 0-1 = \frac{1 \cdot C}{1+1} \Rightarrow C = -2$$

$\therefore$ Eq. (i) reduces to $y-1 = \frac{-2x}{x+1}$

or $\quad (y-1)(x+1) + 2x = 0$