

26. Find the general solution of  $(1 + \tan y)(dx - dy) + 2xdy = 0$ .

Sol. We have,  $(1 + \tan y)(dx - dy) + 2xdy = 0$

$$\Rightarrow (1 + \tan y) \left( \frac{dx}{dy} - 1 \right) + 2x = 0$$

$$\Rightarrow (1 + \tan y) \frac{dx}{dy} - (1 + \tan y) + 2x = 0$$

$$\Rightarrow (1 + \tan y) \frac{dx}{dy} + 2x = (1 + \tan y)$$

$$\Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

This is a linear differential equation.

On comparing it with  $\frac{dx}{dy} + Px = Q$ , we get

$$P = \frac{2}{1 + \tan y}, Q = 1$$

$$\begin{aligned} \text{I.F.} &= e^{\int \frac{2}{1 + \tan y} dy} \\ &= e^{\int \frac{2 \cos y}{\cos y + \sin y} dy} \\ &= e^{\int \frac{\cos y + \sin y + \cos y - \sin y}{\cos y + \sin y} dy} \\ &= e^{\int \left( 1 + \frac{\cos y - \sin y}{\cos y + \sin y} \right) dy} \\ &= e^y + \log(\cos y + \sin y) \\ &= e^y \cdot (\cos y + \sin y) \end{aligned}$$

So, the general solution is:

$$\begin{aligned} x \cdot e^y (\cos y + \sin y) &= \int 1 \cdot e^y (\cos y + \sin y) dy + C \\ \Rightarrow x \cdot e^y (\cos y + \sin y) &= \int e^y (\sin y + \cos y) dy + C \\ \Rightarrow x \cdot e^y (\cos y + \sin y) &= e^y \sin y + C \quad [\because \int e^x (f(x) + f'(x)) dx = e^x f(x)] \\ \Rightarrow x (\sin y + \cos y) &= \sin y + C e^{-y} \end{aligned}$$