15. Find the equation of a curve passing through origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$.

Sol. We have, $(1+x^2)\frac{dy}{dx}+2xy=4x^2$

$$\Rightarrow \qquad \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py=Q$, we get

$$P=\frac{2x}{1+x^2}, Q=\frac{4x^2}{1+x^2}$$

$$\therefore \qquad \text{I.F.}=e^{\int P dx}=e^{\int \frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$$

So, the general solution is:

$$y\cdot(1+x^2)=\int\frac{4x^2}{1+x^2}(1+x^2)dx+C$$

$$\Rightarrow \qquad y\cdot(1+x^2)=\int 4x^2dx+C$$

$$\Rightarrow \qquad y\cdot(1+x^2)=\frac{4x^3}{3}+C \qquad \text{(i)}$$

Since, the curve passes through origin, then putting $x=0$ and $y=0$ in Eq. (i), we get $C=0$

So, the required equation of curve is: $y=\frac{4x^3}{3(1+x^2)}$