14. Form the differential equation* of all circles which pass through origin and whose centres lie on y-axis.

Sol. It is given that circles pass through origin and their centres lie on Y-axis.

Let $(0, k)$ be the centre of the circle and radius is k.

So, the equation of circle is

$$(x-0)^2 + (y-k)^2 = k^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ky = 0 \qquad \text{(i)}$$

Differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0$$

$$\Rightarrow \quad k = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Putting this value of 'k' in Eq. (i), we get

$$x^2 + y^2 - 2y\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}} = 0$$

$$\Rightarrow \quad (x^2 + y^2)\frac{dy}{dx} - 2xy - 2y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$$