4 JEE Main 2016 (Online) 10th April Morning Slot

MCQ (Single Correct Answer)

The solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y},$$

where $0 \leq x < \frac{\pi}{2}$, and $y(0) = 1$, is given by :

A $y = 1 - \frac{x}{\sec x + \tan x}$

B $y^2 = 1 + \frac{x}{\sec x + \tan x}$

C $y^2 = 1 - \frac{x}{\sec x + \tan x}$

D $y = 1 + \frac{x}{\sec x + \tan x}$

Explanation

Given,

$$\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$$

$$\Rightarrow \quad 2y\frac{dy}{dx} + y^2 \sec x = \tan x$$

Now, let

$y^2 = t$

$$\Rightarrow \quad 2y\frac{dy}{dx} = \frac{dt}{dx}$$

$\therefore$ New equation,

$$\frac{dt}{dx} + t\sec x = \tan x$$

$$\therefore \quad \text{I.F} = e^{\int \sec x dx}$$

$$= \quad e^{\ln(\sec x + \tan x)}$$

$=$ sec x + tan x

$\therefore$ Solution is,

$t(\sec x + \tan x) = \int \tan x \, (\sec x + \tan x) \, dx$

$\Rightarrow \quad t(\sec x + \tan x) = \sec x + \tan x - x + c$

$\Rightarrow \quad t = 1 - \frac{x}{\sec x + \tan x} + c$

$\Rightarrow \quad y^2 = 1 - \frac{x}{\sec x + \tan x} + c$

Given,

$y(0) = 1$

$\therefore \quad 1 = 1 - 0 + c$

$\Rightarrow \quad c = 0$

$\therefore \quad y^2 = 1 - \frac{x}{\sec x + \tan x}$