1 **JEE Main 2021 (Online) 1st September Evening Shift**

MCQ (Single Correct Answer)

The function f(x), that satisfies the condition

$f(x) = x + \int_0^{\pi/2} \sin x . \cos y \, f(y) \, dy$, is :

A $x + \frac{2}{3}(\pi - 2)\sin x$

B $x + (\pi + 2)\sin x$

C $x + \frac{\pi}{2}\sin x$

D $x + (\pi - 2)\sin x$

Explanation

$$f(x) = x + \int_0^{\pi/2} \sin x \cos y \, f(y) \, dy$$

$$f(x) = x + sinx \underbrace{\int_0^{\pi/2} \cos y \, f(y) \, dy}_{K}$$

$$\Rightarrow f(x) = x + K \sin x$$

$$\Rightarrow f(y) = y + K \sin y$$

Now, $K = \underset{Apply\ IBP}{\int_0^{\pi/2} y \cos y \, dy} + K \underset{Put\ \sin y = t}{\int_0^{\pi/2} \cos y \sin y \, dy}$

$$K = (y \sin y)_0^{\pi/2} - \int_0^{\pi/2} \sin y dy + K \int_0^1 t \, dt$$

$$\Rightarrow K = \frac{\pi}{2} - 1 + K\left(\frac{1}{2}\right)$$

$$\Rightarrow K = \pi - 2$$