

4 JEE Main 2013 (Offline)

MCQ (Single Correct Answer)

The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x -axis, and lying in the first quadrant is :

A 9

B 36

C 18

D $\frac{27}{4}$

3 JEE Main 2014 (Offline)

MCQ (Single Correct Answer)

The area of the region described by

$A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is :

A $\frac{\pi}{2} - \frac{2}{3}$

B $\frac{\pi}{2} + \frac{2}{3}$

C $\frac{\pi}{2} + \frac{4}{3}$

D $\frac{\pi}{2} - \frac{4}{3}$

Explanation

Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.

Intersection points are $x = 0, 1$

Area of shaded portion is the required area.

So, Required Area = Area of semi-circle + Area bounded by parabola

$$= \frac{\pi r^2}{2} + 2 \int_0^1 \sqrt{1-x} dx$$

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(As radius of circle = 1)

$$= \frac{\pi}{2} + 2 \left[\frac{(1-x)^{3/2}}{-3/2} \right]_0^1$$

Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$, $0 \leq x \leq 1$, and $f(0) = 0$, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

[2009]

C

$$f' = \pm \sqrt{1 - f^2}$$

$$\Rightarrow f(x) = \sin x \text{ or } f(x) = -\sin x$$

$$\Rightarrow f(x) = \sin x$$

$$\text{Also, } x > \sin x \quad \forall x > 0.$$

(not possible)