

Q. No. 9: Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

Solution:

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix} = \hat{i}(8 - 3) - \hat{j}(4 + 1) + \hat{k}(3 + 2) = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-5)^2 + (5)^2} = \sqrt{3(5)^2} = 5\sqrt{3}$$

Hence, the unit vector perpendicular to the plane of \vec{a} and \vec{b} is:

$$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}}$$