Question 8: Let $a = 2i + j - 2k$ and $b = i + j$. If c is a vector such that $a . c = |c|$, $|c - a| = 2\sqrt{2}$ and the angle between $(a \times b)$ and c is 30^o, then $|(a \times b) \times c| =$ ________.

Ans $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} + \hat{j}$

$a \cdot c = |c|$

$|c - a| = 2\sqrt{2}$

$|c-a|^2 = 8$

$c^2 + a^2 - 2\vec{a}\cdot\vec{c} = 8$

$c^2 + 9 - 2\vec{a}\cdot\vec{c} = 8$

$|c|^2 - 2|c| + 1 = 0$

$(|c| - 1)^2 = 0 \quad \Rightarrow \quad |c| = 1$

$|(a \times b) \times c| = |a \times b|\ |c| \sin 30^\circ$

$= \cancel{|a|\,|b|} = |a \times b|\ (1)\ \frac{1}{2}$

$= \frac{|\vec{a} \times \vec{b}|}{2}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(2) - \hat{j}(+2) + \hat{k}($$

$= 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

$\therefore \ |(\vec{a} \times \vec{b}) \times \vec{c}| = \frac{3}{2}$